



PROBLEM #8

Balanced Ternary

INTRODUCTION TO BALANCED TERNARY

- ✗ Uses 1, 0, -1 (written as T) to represent numbers
- ✗ Doesn't need a separate sign for negative numbers
- ✗ Simplest self-contained positional numeral system for integers
 - Binary is the simplest self-contained positional numeral system for natural numbers

I changed the symbols to X, I, V and added 0s to the front of numbers to make all the numbers the same length or it will be too easy



PROBLEM

- ✗ Match the following numbers with the unknown numerals in arbitrary order:
 - $-25, -17, -13, -5, -2, 1, 7, 9, 15, 20$
 - VVVI, VIVV, IXXV, XVIX, VXII, XVI, IXIX, VVXI, VXXX, VIXI
- ✗ Write in Arabic numerals:
 - VVIV, IIXX, XVXX, XIVXX
- ✗ There is actually a shorter way of writing some of the unknown numerals above. Explain.
- ✗ Write in the unknown numerals in shortest form
 - $-4, -35, 23, 100$

SOLUTION



SOLUTION

- ✗ There are only three symbols (X, V, I) so the numerals are probably in ternary (base 3)
- ✗ This means each value is multiplied by 3 raised to the power of the number of symbols to the right
- ✗ Ex: IXVI $\rightarrow 1 * 3^3 + X * 3^2 + V * 3^1 + I * 3^0$
- ✗ There are 5 negative numbers and 5 positive numbers
- ✗ Group the unknown numerals by their starting symbol:
 - VVVI, VIIVV, VXII, VVXI, VXXX, VXI
 - IXXV, IXIX
 - XVIX, XIVI



SOLUTION

- ✗ By looking at question three, guess that one of the symbol is 0 (which can be deleted if it is at the start of the number)
- ✗ First guess that the symbol V is 0
- ✗ Regroup:
 - IXXV, IXIX, VVVI, VIVV, VIXI
 - XVIX, XIVI, VXII, VVXI, VXXX
- ✗ Each group has 5 numbers so one group must be positive while the other one is negative.
- ✗ I(VVVI) is the shortest so it should be 1 \longrightarrow $I = 1$



SOLUTION

- ✗ Now we can solve for the positive numbers:
 - $VIVV \longrightarrow 0 * 27 + 1 * 9 + 0 * 3 + 0 * 1 = 9$
 - $VIXI \longrightarrow 0 * 27 + 1 * 9 + X * 3 + 1 * 1 = 10 + 3X$
 - $IXXV \longrightarrow 1 * 27 + X * 9 + X * 3 + 0 * 1 = 27 + 12X$
 - $IXIX \longrightarrow 1 * 27 + X * 9 + 1 * 3 + X * 1 = 30 + 10X$
- ✗ X must be negative because the largest positive number is 20 (<27, 30)
- ✗ By guessing $X = -1$ we get $VIXI = 7$, $IXXV = 15$, $IXIX = 20$, which matches the problem



SOLUTION

- ✘ Now solve for the negative numbers:
- $XVIX \longrightarrow -1 * 27 + 0 * 9 + 1 * 3 + -1 * 1 = -25$
 - $XIVI \longrightarrow -1 * 27 + 1 * 9 + 0 * 3 + 1 * 1 = -17$
 - $VXII \longrightarrow 0 * 27 + -1 * 9 + 1 * 3 + 1 * 1 = -5$
 - $VVXI \longrightarrow 0 * 27 + 0 * 9 + -1 * 3 + 1 * 1 = -2$
 - $VXXX \longrightarrow 0 * 27 + -1 * 9 + -1 * 3 + -1 * 1 = -13$



SOLUTION

x Question b solution

- $VVIV \longrightarrow 1 * 3 = 3$
- $IIXX \longrightarrow 1 * 27 + 1 * 9 + -1 * 3 + -1 * 1 = 32$
- $XVXX \longrightarrow -1 * 27 + -1 * 3 + -1 * 1 = -31$
- $XIVXX \longrightarrow -1 * 81 + 1 * 27 + -1 * 3 + -1 * 1 = -58$



SOLUTION

x Question d solution

- $-4 = -1 * 3 + -1 * 1 \longrightarrow XX$
- $-35 = -1 * 27 + -1 * 9 + 0 * 3 + 1 \longrightarrow XXVI$
- $23 = 1 * 27 + 0 * 9 + -1 * 3 + -1 * 1 \longrightarrow IVXX$
- $100 = 1 * 81 + 1 * 27 + -1 * 9 + 0 * 3 + 1 * 1 \longrightarrow IIXVI$

THANK YOU